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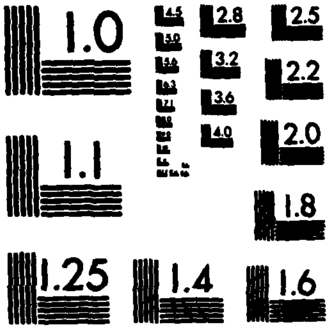

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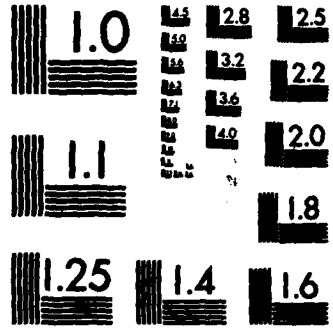

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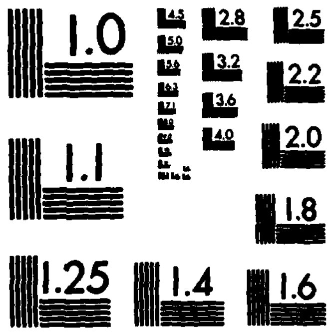

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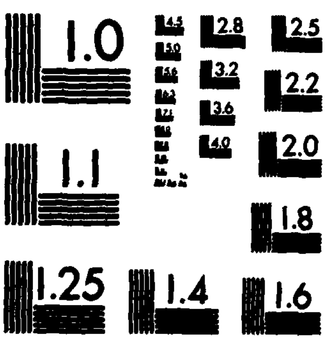

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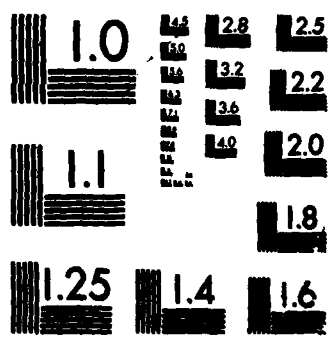

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The contact resistance between the hot wire and the exothermic material is incorporated in the thermal ignition model. A new solution to the interface temperature was developed and used in the explosion criterion to predict more accurately the explosion delay time. The new solution is derived for variable current heating as well as for constant current heating. Use of the solution is illustrated and compared with other solutions of the problem of D.C. heating a wire in perfect contact with exothermic material. It was found that for values of H (the outer

infinity

conductivity of the surface of the wire) between $\cancel{\infty}$ and 0.1, the change in the explosion delay is minimal. By decreasing H further, the explosion delay time increases considerably. Use of the solution for variable current heating is also illustrated in another example.

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INTRODUCTION

The design of improved squibs and detonators depends on better understanding the problem of the initiation of exothermic materials by imbedded hot wires. Existing thermal explosion theory 1/ has been shown to predict experimental hot-wire ignition delay time, but does not show the dependence of ignition delay on the form of heating function and the choice of explosive and wire material. Therefore, the reactivity of the imbedding medium has been characterized by an "ignition temperature" rather than by the more fundamental kinetic parameters of thermal explosion theory. Works 2/ and 3/ based on the concept of the fixed ignition temperature shows that ignition temperature depends on the experimental conditions as well as the chosen wire-explosive system. The ignition temperature of a composite propellant heated by current pulses at preset duration has been investigated 4/ and 5/, and it was found that this explosion property depends on ignition delay and wire diameter.

A different thermal explosion criterion 6/, which incorporates the parametric dependence of the delay time on the form of heating function and the choice of explosive and wire material,has been developed. In developing the theoretical work for this explosion criterion, the system modeled was considered an exothermic material surrounding an infinitely long heated wire of infinite thermal diffusivity and of radius a. The thermal explosion criterion assumes that the nominal explosion site is the wire/explosive interface $r = a$. The interface temperature was an approximation to the solution of a long cylinder in perfect contact with perfect conductor (the wire). The explosion temperature T_e was that value of T which minimizes the function F(T).

$$F(T) = t_{ra}(T) + t_{ad}(T) \tag{1}$$

1/ Mauger, F., The Theory of Thermal Explosions: The Initiation of an Explosion by Hot Wire, ARDE Report No. 9168, 1960.

2/ Altman, D. and Grant, Jr., A. F., Fourth Symposium (International) on Combustion, (Baltimore: Williams and Wilkins, 1953) p. 158.

3/ Kabin, I., Rosenthal, L. A. and Salem, A. D., The Response of Electro-Explosive Devices to Transient Electrical Pulses, NOL TR 61-20, 1961.

4/ Baer, A. D., "Pyrodynamics", 3, 15 (1965).

5/ Cheng, J., Bouch, L. S., Keller, J. A., Baer, A. D. and Ryan, N. W., Ignition and Combustion of Solid Propellants, AFOSR 66-1672, 1965.

6/ Friedman, M. H., "A General Thermal Explosion Criterion Application to Initiation by Imbedded Wires," Combustion & Flame, December 1969.

where $t_m(T)$ is the time at which the interface temperature is T , and t_{ad} is the adiabatic explosion time explained by Friedman 6/. Contrary to practical consideration, the analysis cited above did not consider the contact resistance between the wire and the explosive in calculating the interface temperature. It is expected that due to contact resistance, the interface temperature will lag behind the wire temperature, resulting in longer ignition delays than previously predicted.

Blackwell 7/ calculated the wire temperature caused by heat supplied at constant rate. He also developed a solution to the surrounding medium temperature in the Laplace Transform space. These solutions are in terms of an infinite integral containing complicated expressions of Bessel functions of the first and the second kind. This method was used to develop a more general solution for the temperature-time history of the interface. This solution is valid for variable current heating as well as for constant current heating. It incorporates the parametric dependence of the delay time on the contact resistance between the wire and the explosive. This solution is used with the explosion criterion to more accurately predict ignition delay times.

THEORETICAL ANALYSIS

Assume an infinite region of one material (the explosive) bounded internally by a circular cylinder of another perfectly conducting material (the wire). With the initial and boundary conditions,

a) Zero initial temperature throughout
b) Variable heat input per unit time to the inner cylinder
c) Contact-resistance at the boundary between cylinder and external medium, then

$$\frac{\partial^2 \theta_2}{\partial r^2} + \frac{1}{r}\frac{\partial \theta_2}{\partial r} = \frac{1}{h_2^2}\frac{\partial \theta_2}{\partial t} \qquad b < r < \infty,\ t > 0 \qquad (2)$$

Initial Conditions

$$\theta_1 = \theta_2 = 0 \qquad \text{at} \qquad t = 0 \qquad (3)$$

7/ Blackwell, J. H., "A Transient-Flow Method for Determination of Thermal Constants of Insulating Materials in Bulk," Journal of Applied Physics, Vol. 25, No. 2, 1954.

Boundary Conditions

θ_2 is bounded as $r \to \infty$

$$\left.\begin{aligned} -K_2 \frac{\partial \theta_2}{\partial r} &= H(\theta_1 - \theta_2) \qquad & r = b,\ t > o \\ -K_2 \frac{\partial \theta_2}{\partial r} &= Q'(t) - \frac{\alpha}{2}\frac{\partial \theta_1}{\partial t} \qquad & r = b,\ t > o \end{aligned}\right\} \tag{4}$$

where $\theta_1(t)$ and $\theta_2(t,r)$ are the temperature of the wire and the explosive at any time, and radius r is the radial coordinate, h_2^2 is the diffusivity of the explosive material, K_2 is the thermal conductivity of the explosive material, b is the radius of the wire, t is the time, H is the outer conductivity at the surface $r = b$.

$\alpha = M_1 C_1 / \pi b$ and $Q'(t) = Q(t) / 2\pi b$

where M_1 and C_1 are the mass/unit length and the specific heat of the wire, and Q(t) is the heat supplied/unit wire length/unit time.

Taking Laplace Transform of the differential equation and the boundary conditions results in:

$$\frac{d^2\Theta_2}{dr^2} + \frac{1}{r}\frac{d\Theta_2}{dr} = q^2\Theta_2 \tag{5}$$

$$-K_2 \frac{d\Theta_2}{dr} = H(\Theta_1 - \Theta_2) \qquad r = b \tag{6}$$

$$-K_2 \frac{d\Theta_2}{dr} = Q'(P) - \frac{\alpha P}{2}\Theta_1 \qquad r = b \tag{7}$$

where $\Theta_1(P)$, $\Theta_2(P)$ are the Laplace Transforms of $\theta_1(t)$ and $\theta_2(t,r)$, respectively, p is the transform variable, and $q = \sqrt{P} / h_2$.

The solution of equation 5, subject to the boundary conditions of equations 6 and 7, is

$$\Theta_1(P) = \frac{2bQ'(P)\Delta(b)}{[P\alpha b\Delta(b) + 2K_2K_1(qb)]} \tag{8}$$

$$\Theta_2(P, r) = \frac{2bQ'(P)K_o(qr)}{qb\,[P\alpha b\Delta(b) + 2K_2K_1(qb)]} \tag{9}$$

where $\Delta(r) = \dfrac{K_o(qr)}{qb} + \left(\dfrac{K_2}{bH}\right)K_1(qr)$, $K_o(qr)$, $K_1(qr)$ are modified Bessel functions of the second kind, and zero and first orders respectively, and $\Delta(b)$ is $\Delta(r)$ at $r = b$. Substituting $r = b$ into equation 9 gives the Laplace Transform of the interface temperature, $\Theta_2(P, b)$. $\Theta_1(P)_1$ and $\Theta_2(P)$ can be written as

$$\Theta_1(P) = F_1(P) \cdot Q'(P) \tag{10}$$

$$\Theta_2(P) = F_2(P) \cdot Q'(P) \tag{11}$$

where

$$F_1(P) = \frac{2b\Delta(b)}{[P\alpha b\Delta(b) + 2K_2K_1(qb)]} \tag{12}$$

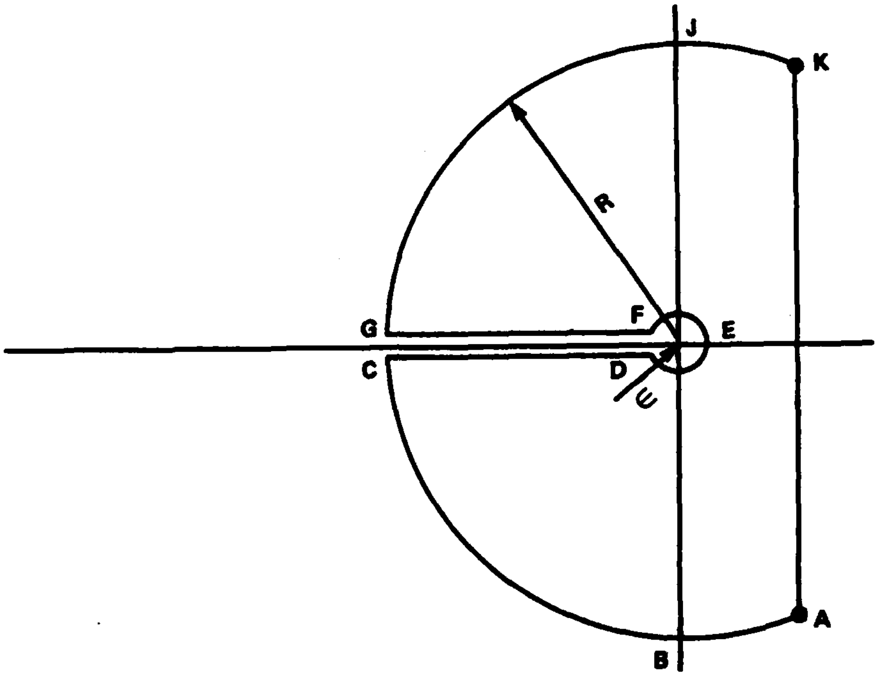
$$F_2(P) = \frac{2bK_o(qb)}{qb\,[P\alpha b\Delta(b) + 2K_2K_1(qb)]} \tag{13}$$

Applying the inversion theorem of the Laplace Transformation, we get

$$f_1(t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} F_1(P)\,e^{+Pt}dp \tag{14}$$

$$f_2(t) = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} F_2(P)\, e^{+Pt} dp \tag{15}$$

The integrands of equations 14 and 15, each has a branch point at the origin, and neither has poles within or on the closed contour AKJGFEDCBA . Denoting this contour by C , we may write



$$\frac{1}{2\pi i}\oint_C F_1(P)\, e^{Pt} dp = \frac{1}{2\pi i}\left\{\int_{AK} F_1(P)\, e^{Pt} dp + \int_{KJG} + \int_{GF} + \int_{FED} + \int_{DC} + \int_{CBA}\right\} \tag{16}$$

$$\frac{1}{2\pi i}\oint_C F_2(P)\, e^{Pt} dp = \frac{1}{2\pi i}\left\{\int_{AK} F_2(P)\, e^{Pt} dp + \int_{KJG} + \int_{GF} + \int_{FED} + \int_{DC} + \int_{CBA}\right\} \tag{17}$$

By Cauchy's Theorem, the integrals on the left side of equations 16 and 17 are zeros. As the radius R of the outside circular arc $\rightarrow \infty$, and as the radius ϵ of the inside circular arc $\rightarrow 0$, the second, the fourth, and the sixth integrals of the right side of each, equations 16 and 17 vanish. Thus

$$f_1(t) = \frac{1}{2\pi i} \lim_{\substack{R \to \infty \\ \epsilon \to 0}} \left\{ \int_{FG} F_1(P)\, e^{Pt}\, dp + \int_{CD} F_1(P)\, e^{Pt}\, dp \right\} \tag{18}$$

$$f_2(t) = \frac{1}{2\pi i} \lim_{\substack{R \to \infty \\ \epsilon \to 0}} \left\{ \int_{FG} F_2(P)\, e^{Pt}\, dp + \int_{CD} F_2(P)\, e^{Pt}\, dp \right\} \tag{19}$$

The following substitutions are made in equations 18 and 19: $P = \sigma e^{-\pi i}$ on CD , and $P = \sigma e^{\pi i}$ on FG

$$f_1(t) = \frac{1}{2\pi i} \left\{ \int_0^\infty F_1(\sigma e^{-i\pi})\, e^{-\sigma t}\, d\sigma - \int_0^\infty F_1(\sigma e^{i\pi})\, e^{-\sigma t}\, d\sigma \right\} \tag{20}$$

$$f_2(t) = \frac{1}{2\pi i} \left\{ \int_0^\infty F_2(\sigma e^{-i\pi})\, e^{-\sigma t}\, d\sigma - \int_0^\infty F_2(\sigma e^{i\pi})\, e^{-\sigma t}\, d\sigma \right\} \tag{21}$$

Reduction of equations 20 and 21 entails replacement of modified Bessel functions of imaginary argument by ordinary Bessel functions of real arguments. Following Blackwell, the substitution $x = \sqrt{\sigma}/h_2$ is then made for convenience. We obtain

$$f_1(t) = \frac{16\, K_2}{\alpha^2 b \pi^2 h_2^2} \int_0^\infty \frac{e^{-x^2 h_2^2 t}}{x\, [P_1^2 + Q_1^2]}\, dx \tag{22}$$

$$f_2(t) = \frac{16\, K_2}{\alpha^2 b \pi^2 h_2^2} \int_0^\infty \frac{\left(1 - \frac{\alpha h_2^2 x^2}{2H}\right) e^{-x^2 h_2^2 t}}{x\, [P_1^2 + Q_1^2]}\, dx \tag{23}$$

where

$$P_1 = x J_0(bx) + K_2 J_1(bx)\left\{\frac{1}{H} x^2 - \frac{2}{\alpha h_2^2}\right\} , \qquad (24)$$

$$Q_1 = x Y_0(bx) + K_2 Y_1(bx)\left\{\frac{1}{H} x^2 - \frac{2}{\alpha h_2^2}\right\} \qquad (25)$$

where $J_0(bx)$, $J_1(bx)$ are Bessel functions of first kind of zero and first order respectively, and $Y_0(bx)$, $Y_1(bx)$ are Bessel functions of the second kind of zero and first order, respectively. Equations 10 and 11 show that the Laplace Transform of the wire temperature and the interface temperature are the product of two transforms each $F_1(P) \cdot Q'(P)$, and $F_2(P) \cdot Q'(P)$, respectively. A convolution solution is possible in principle.

$$\theta_1(t) = \int_0^t f_1(u)\, Q'(t-u)\, du \qquad (26)$$

$$\theta_2(t) = \int_0^t f_2(u)\, Q'(t-u)\, du \qquad (27)$$

substituting equations 22 and 23 into equations 26 and 27, we get

$$\theta_1(t) = \frac{16 K_2}{\alpha^2 b \pi^2 h_2^2} \int_0^t \int_0^\infty \frac{e^{-x^2 h_2^2 u}\, Q'(t-u)\, dx}{x\,[P_1^2 + Q_1^2]}\, du \qquad (28)$$

$$\theta_2(t_1 b) = \frac{16 K_2}{\alpha^2 b \pi^2 h_2^2} \int_0^t \int_0^\infty \frac{\left(1 - \frac{\alpha h_2^2 x^2}{2H}\right) e^{-x^2 h_2^2 u}\, Q'(t-u)\, dx}{x\,[P_1^2 + Q_1^2]}\, du \qquad (29)$$

The solution for constant current heating may be obtained by replacing $Q'(t)$ in equations 28 and 29 by the constant value Q', and perform the integration with respect to the dummy variable u.

$$\theta_1(t) = \frac{16 K_2 Q'}{\alpha^2 b \pi^2 h_2^4} \int_0^\infty \frac{(1 - e^{-x^2 h_2^2 t})}{x^3 [P_1^2 + Q_1^2]} dx \tag{30}$$

$$\theta_2(t_1 b) = \frac{16 K_2 Q'}{\alpha^2 b \pi^2 h_2^4} \int_0^\infty \frac{\left(1 - \frac{\alpha h_2^2 x^2}{2H}\right)\left(1 - e^{-x^2 h_2^2 t}\right)}{x^3 [P_1^2 + Q_1^2]} dx \tag{31}$$

Equation 30 is the solution derived by Blackwell 7/ for the probe temperature. For zero contact-resistance between the wire and the explosive, the solution may be obtained by letting $H \to \infty$ in equations 30 and 31. In such case the wire temperature and the interface temperature will be equal.

$$\theta_1(t) = \theta_2(t_1 b) = \frac{16 K_2 Q'}{\alpha^2 b \pi^2 h_2^4} \int_0^\infty \frac{\left(1 - e^{-x^2 h_2^2 t}\right)}{x^3 \lim\limits_{H \to \infty} [P_1^2 + Q_1^2]} dx \tag{32}$$

where $$\lim_{H \to \infty} P_1 = x J_0(bx) - \frac{2 K_2}{\alpha h_2^2} J_1(bx)$$

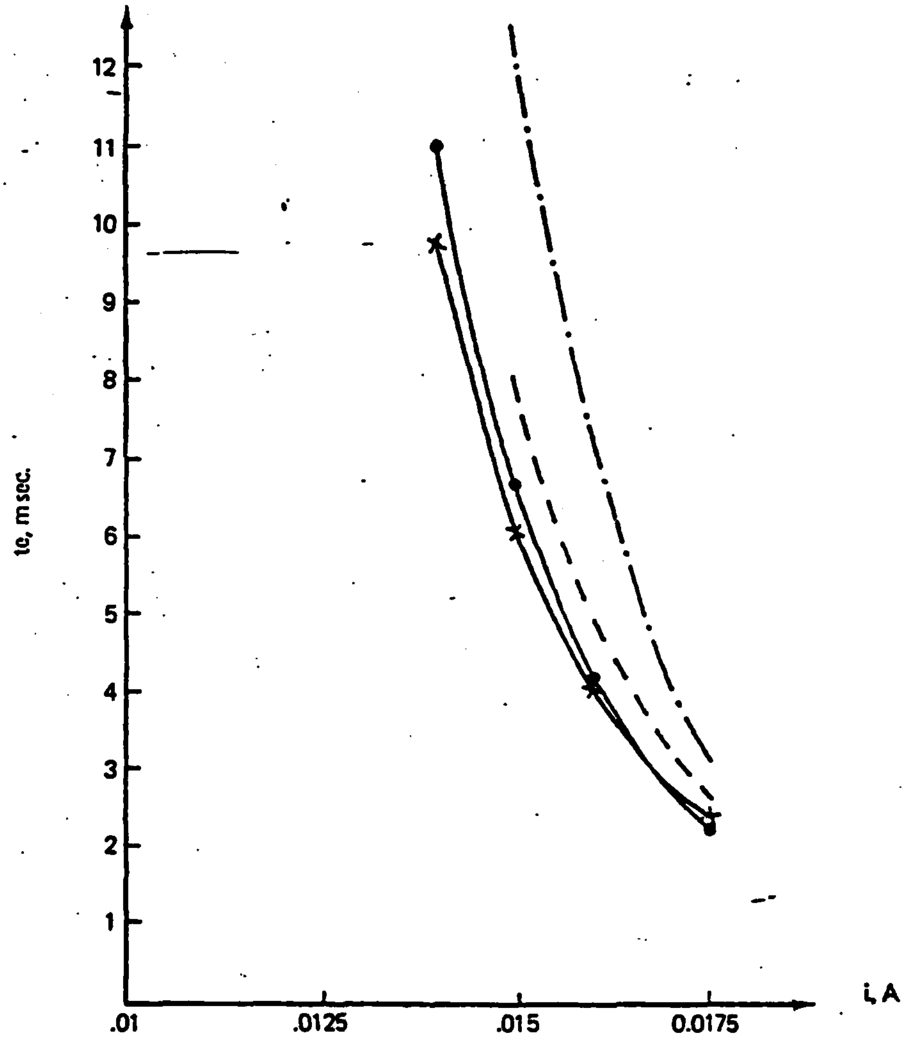
and $$\lim_{H \to \infty} Q_1 = x Y_0(bx) - \frac{2 K_2}{\alpha h_2^2} Y_1(bx)$$

By proper change of variables, equation 32 reduces to T_a given by Friedman 6/. Equation 29 may be integrated numerically to provide a general solution for the interface temperature. Equation 29, together with equation 1, may be used to predict a more accurate ignition delay time.

EXAMPLES

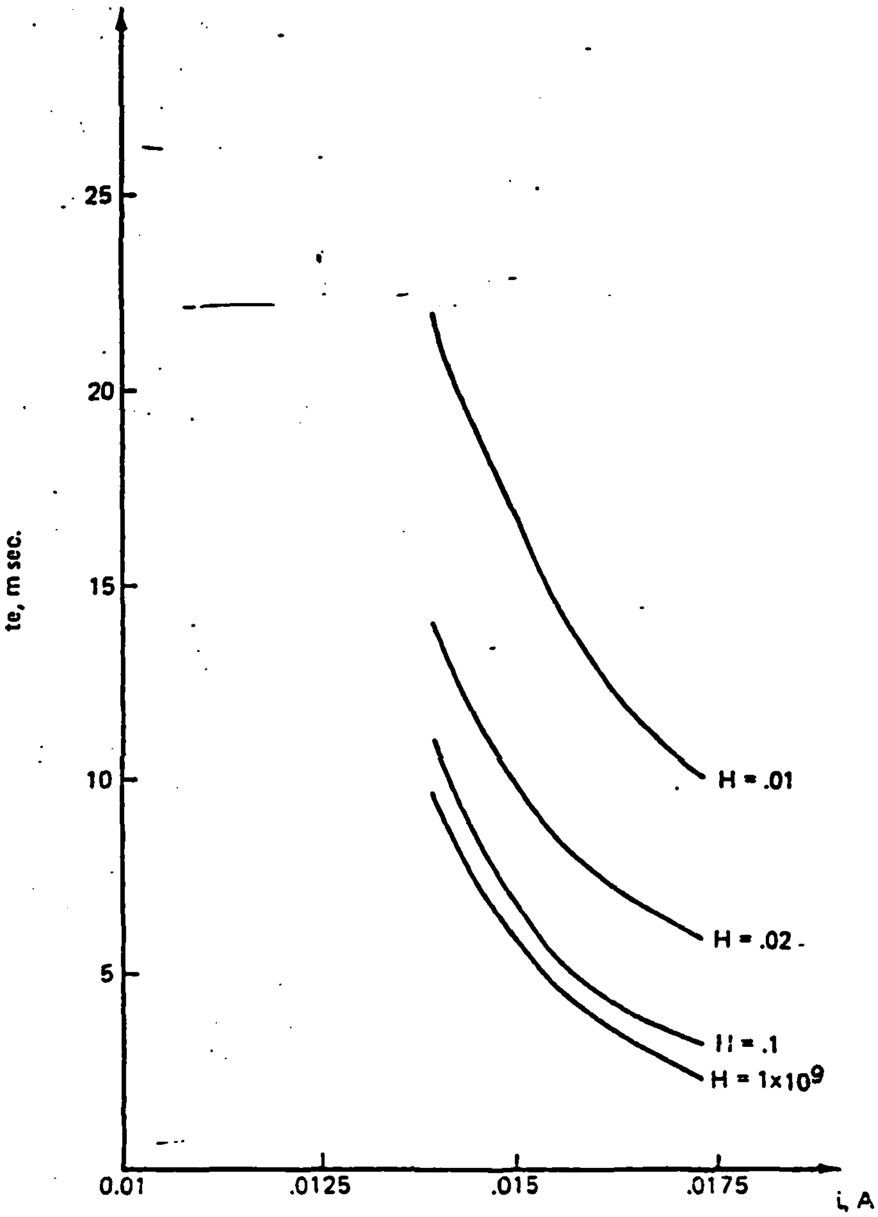
1. A 0.1 mil diameter Nichrome wire in lead styphnate. Figure 1 shows a comparison of predicted explosion delays with those obtained from computer experiments. The solution from equation 31 for constant current heating agrees very well with the exact and quadratic solutions 6/. Figure 2 shows the effect of the contact resistance on the predicted explosion delays of the same wire-explosive system. For values of H , the outer conductivity of the surface between 0.1 and ∞ , the change in the explosion delay is small, but decreasing H further results in considerable increase in the explosion delay.

2. Heating function $i(t) = 0.323 \sin(314560t)$. This corresponds to a 0.1 mil diameter Nichrome wire in lead styphnate, the temperature-time history calculated from integrating equation 30 numerically is compared with the exact solution. The explosion time from equations 29 and 1 was found to be 2.75 μsec. The explosion time found by numerically integrating the full set of differential equations, including chemical reaction, is 2.74 μsec. The solution presented in figure 3 is in agreement with those previously published.



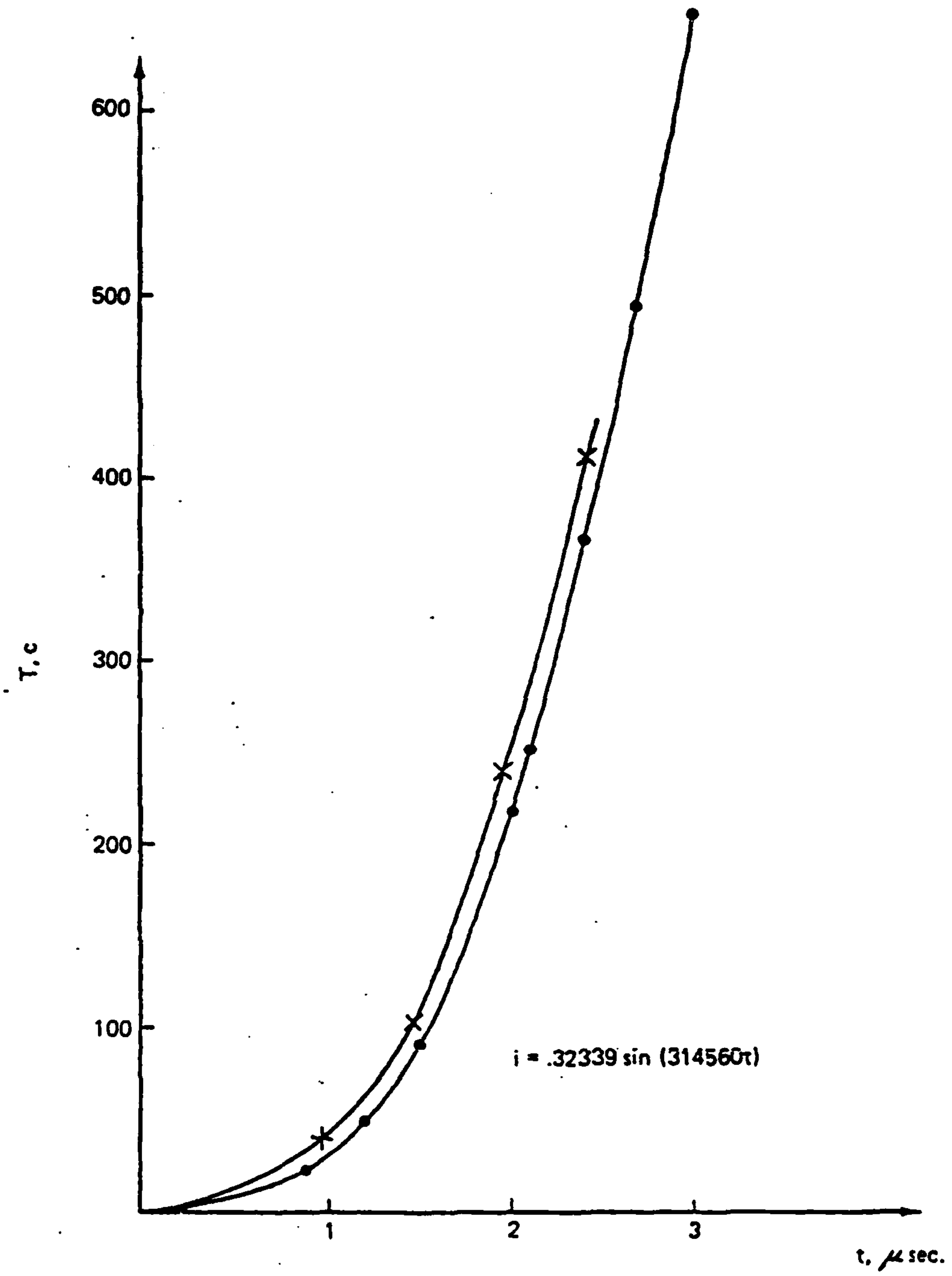
Comparison of predicted explosion delays with those obtained from computer experiments, 0.1 mil diameter Nichrome wires in lead styphnate - - -, Mauger's experiments: ———, exact and quadratic solutions: ——— • ———, Friedmans' approximate solution: X———X, Equations (30) and (31) with H = 1E9

Figure 1. Comparison of predicted explosion delays with those obtained from computer experiments



Comparison of predicted explosion delays obtained from equations (30) and (31) (D.C. current) for different values of H, the outer conductivity at the surface.

Figure 2. Effects of contact resistance on predicted explosion delays



Comparison of temperature-time history of surface of lead styphnate due to sinusoidal current in 0.1 mil diameter Nichrome wire: ———— exact solution: X———X, equations (28) and (29) with H = 1E9.

Figure 3. Temperature-time history at interface of explosive and wire due to sinusoidal current

GLOSSARY OF TERMS

b	=	Radius of wire
C_1	=	Specific heat of wire
$f(t)$	=	Function of real time t
$F(T)$	=	Function of temperature T
$\bar{F}(P)$	=	Function of transform variable P
h_2^2	=	Diffusivity of the explosive material
H	=	The outer conductivity at the interface ($r=b$)
$i(t)$	=	Heating current function
$J_0(\ )$	=	Bessel function of first kind, zero order
$J_1(\ )$	=	Bessel function of first kind, first order
$K_0(\ )$	=	Modified Bessel function of second kind, zero order
$K_1(\ )$	=	Modified Bessel function of second kind, first order
M_1	=	Mass of wire/unit length
p	=	Transform variable
P_1	=	Equation 24
q	=	$\sqrt{P}/h_2$
$Q(t)$	=	Heat supplied/unit wire length/unit time
$Q'(t)$	=	$Q(t)/2\pi b$
$\bar{Q}(p)$	=	Function of P
Q_1	=	Equation 25
r	=	Radial coordinate
R	=	Outside radius of circle of integration
t	=	Time
$t_{ra}(T)$	=	Time at which the interface temperature equal T
$t_{ad}(T)$	=	Adiabatic explosion time
T	=	Interface temperature

u	=	Dummy variable
X	=	$\sqrt{\sigma}/h_2$
$Y_0(\ )$	=	Bessel function of second kind, zero order
$Y_1(\ )$	=	Bessel function of second kind, first order
α	=	$M_1 C_1/\pi b$
$\Delta(r)$	=	Function of r
$\Delta(p)$	=	Function of p
ϵ	=	Inside radius of integration circle
γ	=	Dummy variable
$\Theta_1(t)$	=	Temperature of wire function of time
$\Theta_2(t,r)$	=	Temperature of explosive function of time and radius
Θ	=	Laplace transform of temperature
σ	=	Dummy variable